IBMM: Data Processing Model for Reciprocal Pairwise Comparison Matrix

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Extended Abstract

The reciprocal pairwise comparison matrix (RPCM) is a well established technique and widely used in multiple criteria decision making (MCDM) methods to perform pairwise comparisons and derive the weight vectors of items, especially in the AHP/ANP. The RPCM is one of the key components in the AHP/ANP which are used to quantify the qualitative and/or intangible attributes into measurable numerical values. To make a valid and reliable decision, there are several issues need to be handled before a RPCM is used to derive the weight vectors and make the final decision. First, the RPCM must be complete, otherwise, the missing value(s) should be estimated, and second, the RPCM should pass the consistency test, otherwise, the inconsistent element(s) should also be identified and adjusted. However, some entries in a RPCM may not be available in many real-world decision problems. Some of the comparison matrices are also inconsistent as the limited expertise, preference conflict as well as the complexity nature of the decision problems etc. Besides, adding or deleting a new criterion or alternative, the rank reversal may occur, which shows that either missing one criterion/alternative or adding one more criterion/alternative in the process of decision making may cause the invalid or unreliable decision-making.

Therefore, it is necessary and meaningful to propose a data processing model to deal with the above three issues for a RPCM. In this talk, an induced bias matrix model (IBMM), which combines the matrix multiplication and the properties of the original RPCM, is proposed to process the data in the RPCM. For the inconsistent RPCM, a bias matrix, based on the original RPCM, is induced to identify the inconsistent elements through analyzing the largest data in the bias matrix. For the incomplete RPCM, the IBMM is used to estimate the missing scores by minimizing all bias values of the induced bias matrix (IBM). The missing value(s) can be estimated by solving a system of equations from the bias induced matrix whilst keeping the global consistency.

There are two main situations which may cause rank reversal, that is, adding new alternatives/criteria or deleting old ones. Rank reversals caused by the above reasons were attributed to the use of relative measurement and normalization. However, there are few data processing models which can be used to perform the sensitivity of rank reversal or point out the critical values of rank reversal when new criterion is added or old one deleted. The IBMM is used to explore and analyze the sensitivity of rank reversal and provide a guideline for determining the critical values of rank reversal. Without losing generality, we focus on the case that a new criterion or alternative is added. The specific

processes of data processing are as follows. First, add a new row or column with unknown variables such as x, y, z etc to the existing RPCM and obtain a new revised RPCM when a new criterion or alternative is added. Second, apply the IBMM to this revised RPCM, and then we can get a system of relation equations about the unknown variables. Third, optimize the system of relation equations by combining with the constraint condition of 9-point scale, and the range of unknown variable values can be estimated. Finally, perform the sensitivity and obtain the critical values of rank reversal.

In summery, the IBMM, as a data processing model, is proposed to identify the inconsistent element(s) of RPCM and estimate the missing values of RPCM. Besides, the IBMM can also be used to analyze the sensitivity of rank reversal and estimate the critical values of rank reversal.

Keywords: Induced bias matrix model (IBMM), Data processing, reciprocal pairwise comparison matrix (RPCM), Inconsistency identification, Missing values estimation, AHP/ANP